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Stability of Compliant Planar Robotics Grasp, with Application to Fruits Grasping.

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Abstract

Robotics grasps and Grasping mechanisms (*grippers*) are used for a wide range of applications including fixturing arrangements, industrial, agricultural, and services robotics in medical and home usage. The gripper's goal is to immobilize the object it is manipulating while applying the minimal necessary grasping effort. Applying too much of the grasping effort may damage the grasped object.

A feasible robotics grasp should be in equilibrium, stable, and robust. While grasp robustness and equilibrium were studied in many researches and works, the grasp's stability was not given much attention. Dynamic stability defines the object's ability to remain grasped under local state perturbations. The reason it was less studied is that it involves the complex contact mechanics between the robotics fingers and the grasped object. In this work we imply a compliant contact model which allows us to analytically develop the full dynamic equations of motion of the combined system – of the object and the gripper. Grasp model includes the kinematic, dynamic, and *contact model*; these respectively describe the system's mechanical constraints, dynamic behavior of the system's elements, and the coupling between the wrench reaction due to displacements at the object - gripper interface.

Motivated by the grasping arrangement of a robotic gripper grasping a fruit during harvesting operation, a complete grasp system model is derived. It is composed of the grasped object's and the gripper's dynamics and embedded with *compliant contact model*. This compliance is essential when considering the grasp of non-rigid objects such as agricultural produce, and therefore plays a major role in the grasp analysis. A Hertz based contact model is considered with added frictional compliance in the tangential direction. We show that due to the asymmetry of the contact model in which the normal force depends on the normal deflection, while the tangential force depends both on the normal and tangential deflection, the overall system is asymmetric. Therefore we present a new stability criterion for asymmetric systems. Hence, a grasp *stability criterion* is developed based on the complete dynamic's symmetry characteristics. The criterion guarantees grasp stability of the whole grasp system. It is further shown that, the new derived grasp model comprises and extends existing known grasp models. The performed analysis is shown to match different types of grasps such as fingertip grasp and *whole arm grasping*. Finally simulation results of the complete grasping arrangement are shown and the stability properties of the grasp are investigated numerically in simulation to match the analytical stability results.
The outcome of this work are set of rules which help to synthesize grasps which are dynamically stable, and which will contribute to fruits' grasp planning during harvesting.

Keywords: Grasp dynamic stability, asymmetric systems, compliant grasps

1. Introduction

Research Objective: Define a grasp stability criterion, for the grasping of deformable objects – that will be used for grasp synthesis and planning, and as a gripper design objectives.

Robots are commonly utilized for the "*four D's*" – dull, dirty, dangerous, and distant chores. For processes including an object's grasping and manipulation, a *grasp synthesis* is needed. Grasp synthesis is a process in which a proper grasp is selected out of many possible grasps. The synthesis is governed by a *grasp quality criterion*, usually dependent on the *grasp analysis* of parameters such as the grasp's equilibrium, robustness, stability, dexterity, precision, approach capabilities, and the damage the object or the gripper may suffer. These grasp's properties are directly influenced by the object's and the grasping mechanism (*gripper*) material properties, geometry, actuation and sensing accuracy, and environmental influences. Having that said, adding a stability criterion to the other grasp quality criteria and parameters (Ohev Zion&Shapiro,2011), is crucial for achieving a desired optimized grasp.
In many grasping applications, the *grasp system* (an object grasped by a gripper) cannot be analyzed as rigid, *e.g.* manipulating agriculture produce, food products, biological organisms, and compliant or fragile materials. In these cases the interface between the grasped object and the gripper has to be modeled (*contact model*) and analyzed as compliant. Although analyzing a grasp system embedded with a compliant contact model provides tools to handle some rigid contact model phenomena, it imposes new and complex problems. These complexities are within the kinematic model with its multi *DoF* (degrees of freedom) dependencies, followed by the dynamic response and characteristics that are highly influenced by the contact model.

2. Stability of Anti—Symmetric Second Order Dynamic System

Let us first define the measure for anti-symmetric second order dynamic system. This measure extends the work in (Shapiro, 2005) (Shapiro, et al., 2013), and will be later implemented to the derived grasp model.
Definitions 2.1: *Spectral norm of a matrix* $M \in \mathbb{R}^{n\times n}$ *, whose eigenvalues are* $\lambda(M) \in \mathbb{C}$ *, is:*

$$\|M\| = \max\left\{\sqrt{\lambda(M^T M)}\right\} \tag{2.1}$$

Definitions2.2: *The symmetric and anti-symmetric parts are of a matrix* $M \in \mathbb{R}^{n\times n}$ *, are:*

$$M_s = \frac{1}{2}\left(M + M^T\right) \quad , \quad M_{as} = \frac{1}{2}\left(M - M^T\right), \quad ; \quad M = M_{as} + M_s \tag{2.2}$$

Theorem 2.1: *For a vector* $v \in \mathbb{C}^n$ *with its conjugate transpose* $v^\dagger$ *, together with definitions 2.1, 2.2 and the symmetric and anti-symmetric matrices' properties, it is possible to show that*

$$\begin{gathered} \|v\|^2 \min\left(Im\left(\lambda\left(M_{as}\right)\right)\right) \le Im\left(v^\dagger M_{as}\ v\right) \le \|v\|^2 \max\left(Im\left(\lambda\left(M_{as}\right)\right)\right), \\ \|v\|^2 \min\left\{\lambda\left(M_s\right)\right\} \le v^\dagger M_s v \le \|v\|^2 \max\left\{\lambda\left(M_s\right)\right\} \end{gathered} \tag{2.3}$$

The proof was relegated to (Ohev-Zion, 2015). ■
Theorem 3.2: *Consider the linear system of the form*

$$\dot{\xi}(t) = \begin{bmatrix} 0_{n\times n} & I_n \\ -D^{-1}Kp & -D^{-1}Kd \end{bmatrix}\left(\xi(t) - \xi^*\right) = A\left(\xi(t) - \xi(t)^*\right); \{Kp_, Kd\} \in \mathbb{R}^{n\times n}, \tag{2.4}$$

with $D = I_n$ *that is the identity matrix, and* ξ^* its equilibrium configuration. *Let the symmetric and anti-symmetric parts of the stiffness and damping matrices* Kp, Kd *be as described in definition 2.1. The linear system (2.4) is globally exponentially stable, if*

$$\begin{gathered} \gamma^2 < \beta\left(\alpha\beta + \gamma\epsilon\right) \quad ; \\ \alpha = \lambda_{min}\left(Kp_s\right) > 0 \quad , \quad \beta = \lambda_{min}\left(Kd_s\right) > 0 \quad , \quad \gamma = -\|Kp_{as}\| \quad , \quad \epsilon = \|Kd_s\| \end{gathered} \tag{2.5}$$

Proof: Consider the linear system as in(2.4), with $D(\xi^*) = I_n$ that is identity matrix. Let $\lambda(A) \in \mathbb{C}$ be an eigenvalue of the matrix A, with its corresponding eigenvector $v = (v_1, v_2)^T \in \mathbb{C}^n$. Without the loss of generality it is possible to take $\|v_1\| = 1$. By definition

$$\begin{pmatrix} 0 & I \\ -Kp & -Kd \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_2 \\ -Kpv_1 - Kdv_2 \end{pmatrix}, \tag{2.6}$$

and the following substitutions are possible for λ^2

$$\begin{aligned} \lambda^2 &= v_1^\dagger \lambda^2 v_1 = v_1^\dagger \lambda v_2 = v_1^\dagger\left(-Kpv_1 - Kdv_2\right) = v_1^\dagger\left(-Kpv_1 - \lambda Kdv_1\right) \\ &= -v_1^\dagger Kp_s v_1 - v_1^\dagger Kp_{as} v_1 - \lambda v_1^\dagger Kd_s v_1 - \lambda v_1^\dagger Kd_{as} v_1. \end{aligned} \tag{2.7}$$

Substituting the following scalars, and (2.7) becomes

$$\begin{gathered} \tilde{\alpha} = v_1^\dagger Kp_s v_1, \quad i\tilde{\gamma} = v_1^\dagger Kp_{as} v_1, \quad \tilde{\beta} = v_1^\dagger Kd_s v_1, \quad i\tilde{\epsilon} = v_1^\dagger Kd_{as} v_1 \quad ;\{\tilde{\alpha},\tilde{\beta},\tilde{\gamma},\tilde{\epsilon}\} \in \mathbb{R} \\ \lambda^2 + \lambda(\tilde{\beta} + i\tilde{\epsilon}) + \tilde{\alpha} + i\tilde{\gamma} = 0, \end{gathered} \tag{2.8}$$

which is quadratic and obtain the known solutions that satisfies every λ

$$\lambda_{1,2} = \frac{-(\tilde{\beta} + i\tilde{\epsilon}) \pm \sqrt{(\tilde{\beta} + i\tilde{\epsilon})^2 - 4(\tilde{\alpha} + i\widetilde{\gamma})}}{2}. \tag{2.9}$$

As known from linear system theory, system(2.4) is exponentially stable if all of A's eigenvalues satisfied $Re(\lambda(A)) < 0$. State this criterion in terms of (2.9) to obtain

$$Re(\lambda_{1,2}) = Re\left(\frac{1}{2}\left(-(\tilde{\beta} + i\tilde{\epsilon}) \pm \sqrt{(\tilde{\beta}^2 - \tilde{\epsilon}^2 - 4\tilde{\alpha}) + i(2\tilde{\beta}\tilde{\epsilon} + \tilde{\gamma})}\right)\right) < 0. \tag{2.10}$$

With $\tilde{\alpha} > 0$ and $\tilde{\beta} > 0$, (2.10) is reduced to

$$\tilde{\gamma}^2 < \tilde{\beta}\left(\tilde{\alpha}\tilde{\beta} + \tilde{\gamma}\tilde{\epsilon}\right). \tag{2.11}$$

These four parameters are quadratic forms(2.8), and their magnitudes are bounds(2.3). In order to obtain a conservative criterion, consider the minimal values of α, β, γ , and the maximal value of ϵ - so the argument $\tilde{\gamma}\tilde{\epsilon} \leq 0$. It is worth reminding that γ and ϵ are bounded by the eigenvalues of skew-symmetric matrices, and are conjugate paired. Therefore criterion (2.11) is bounds by

$$\left(\lambda_{\min}\left(Kp_{as}\right)\right)^2 < \lambda_{\min}\left(Kd_s\right)\left(\lambda_{\min}\left(Kp_s\right)\lambda_{\min}\left(Kd_s\right) + \lambda_{\min}\left(Kp_{as}\right)\lambda_{\max}\left(Kp_{as}\right)\right), \tag{2.12}$$

which is corresponding with Lemma's 2.1

$$\gamma^2 < \beta\left(\alpha\beta + \gamma\epsilon\right)$$ ■

Resolving the theorem's 3.1 assumption of $D(\xi^*) = I_n$, is the same as in (Shapiro, 2005).

3. Kinematic Model

Consider a planar object with its fixed frame $\mathcal{B} = \{\hat{x}_b, \hat{y}_b\}$, whose translation and rotation relative to the world inertial frame $W = \{\hat{x}_0, \hat{y}_0\}$ is described by the time dependent parameters vector $q_0 = (d_0^T, \theta_0)^T = (d_x(t), d_y(t), \theta_0(t))^T \in \mathbb{R}^3$. $\mathcal{B}$'s shape is known with respect to $\mathcal{B}$, therefore each of the i'th nominal contact point's radius of curvature $rc_{x,i}$ and its center of curvature location $r_{cc0,i}$, are also known with respect to $\mathcal{B}$. With respect to W the i'th contact point center of curvature is defined by $x_{cc,i} = d_0 + R_{\theta_0} r_{cc,0,i}$, where $R_{\theta_0} \in SO(2)$ is the object's rotation matrix. The object is grasped by a gripper through k compliant contact points, with their local center of curvature location $y_{cc,i}(q_i) \in \mathbb{R}^2$ and radius of curvature $rc_{y,i}$, Figure 1(a). The parameters vector $q_i = (q_{i,1}(t), \ldots, q_{i,n_i}(t))^T \in \mathbb{R}^{n_i}$ is a part of the gripper's configuration parameters vector q_g . In order to suits varied gripper topologies, which in some several DoF may be common to multiple contact point, q_g is defined by $q_g = \{\cup q_i\} \in \mathbb{R}^{n_g}$. The system's configuration parameters vector is therefore $q = (q_0^T, q_g^T)^T \in \mathbb{R}^n; n = n_0 + n_g$. In order to maintain the short notation, the time dependent notation (t) will be further discarded.

All the bodies are regarded as rigid, yet compliant at the vicinity of the contact points. The compliant contact model defines the relation between the compliance and forces at the contact points. Local deformations of the contacting bodies are encapsulate by considering the un-deformed objects as virtually penetrating each other – or sharing an overlap segment (Rimon, et al., 2006) (Shapiro, et al., 2013). The overlap segment of the i'th contact point is defined by the distance of its endpoints x_i, y_i - for the object and the gripper respectively, which are the two bodies' local centers of curvatures relative deflections functions. The following relations are therefore $rc_{x,i} = \|x_i - x_{cc,i}\| = const$, $rc_{y,i} = \|y_i - y_{cc,i}\| = const$. These endpoints are defined, where the line connecting the object's and the gripper's centers of curvature coincide with the virtually un-deformed penetrated perimeters. It follows that their locations change along the *load trajectory*, as the two centers of curvature are relatively deflects.

Definition 3.1: *The* 2×2 *skew-symmetric (anti-symmetric) matrix* $\hat{J}$ *is,* $\hat{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Based on all the definitions above, it is obvious that both x_i and y_i are dependent on the whole q, and not separately on q_0 or q_g, Figure 1(b). Therefore the object's grasp matrix G and the gripper's Jacobean matrix J are the sums of their respectively local matrices

$$G = \sum_{i=1}^{k} G_i = \sum_{i=1}^{k} \frac{dx_i}{dq} = \sum_{i=1}^{k} \frac{d}{dq}\left(\mathrm{x}_{cc,i} - \hat{\mathrm{n}}_i \mathrm{rc}_{x,i}\right) = \sum_{i=1}^{k} \left[\frac{d\mathrm{x}_{cc,i}}{dq_0}, 0_{3\times n_g}\right] - \mathrm{rc}_{x,i}\frac{d\hat{\mathrm{n}}_i}{dq} = \sum_{i=1}^{k}\left[G^T{}_{cc,i}, 0_{3\times n_g}\right] - \mathrm{rc}_{x,i}\frac{d\hat{\mathrm{n}}_i}{dq}$$

$$J = \sum_{i=1}^{k} J_i = \sum_{i=1}^{k} \frac{dy_i}{dq} = \sum_{i=1}^{k} \frac{d}{dq}\left(\mathrm{y}_{cc,i} + \hat{\mathrm{n}}_i \mathrm{rc}_{y,i}\right) = \sum_{i=1}^{k} \left[0_{3\times n_0}, \frac{d\mathrm{y}_{cc,i}}{dq_g}\right] + \mathrm{rc}_{y,i}\frac{d\hat{\mathrm{n}}_i}{dq} = \sum_{i=1}^{k}\left[0_{3\times n_0}, J_{cc,i}\right] + \mathrm{rc}_{y,i}\frac{d\hat{\mathrm{n}}_i}{dq}$$

Here $\hat{n}_i$ is the inward pointing normal vector at the i'th contact point - defined by the object's curvature, and since $\|x_i - y_i\| << \|x_{cc,i} - y_{cc,i}\|$, it can further be defined as

$$\hat{n}_i \cong \frac{y_i - x_i}{\|y_i - x_i\|} = \frac{x_{cc} - y_{cc}}{\|x_{cc} - y_{cc}\|}.$$

Correspondingly the tangential unit vector is $\hat{t} = \hat{J}\hat{n}$. This generic model is independent on the gripper's or the grasp's configuration, and further applicable for *fingertips grasping*, *a whole arm grasp*, and configuration in which some "fingers" may have multiple contact point with the object. As long as the object and the gripper are in contact, the penetration along the normal direction is a positive scalar $\delta_{i,n} = \|y_i - x_i\| > 0 \in \mathbb{R}$. The change in the normal overlap segment defines the change of the tangential one so that $\delta_{i,t} = \int \dot{\delta}_{i,t} dt$ and

$$\frac{d\delta_{i,n}}{dt} = -\hat{n}_i^T\left[G_{cc,i}^T - J_{cc,i}\right]\dot{q} \quad \Rightarrow \quad \frac{d\delta_{i,t}}{dt} = \frac{\delta_{i,n}}{\|x_{cc,i} - y_{cc,i}\|}\hat{n}_i^T \hat{J}^T \left[G_{cc,i}^T - J_{cc,i}\right]\dot{q} = \delta_{i,n}\hat{n}_i^T \hat{J}^T \dot{\hat{n}}_i$$

4. Contact Model

A compliant contact model defines a correlation between the system's displacements to the force reaction at the contact point. Here it constraints the grasped object's and the gripper's dynamics, and is further considered as local i.e. the deformations are at the contact point's vicinity only. The derived grasp model is compatible with any contact model of the form

$$fcm_i = \begin{pmatrix} fcm_{i,n} \\ fcm_{i,t} \end{pmatrix} = \begin{pmatrix} g_{n,i} + \phi_{n,i} \\ g_{t,i} + \phi_{t,i} \end{pmatrix}; \quad fcm_i = 0 \quad \forall \quad \delta_{i,n} \le 0 \quad Fc_i = R_i\, fcm_i \quad ; \quad Fc = [Fc_1^T, \ldots, Fc_k^T]^T$$

Where the functions $g_{n,i} = g_{n,i}\left(\delta_{i,n}(q)\right), \phi_{n,i} = \phi_{n,i}\left(\delta_{i,n}(q), \dot{\delta}_{i,n}(q,\dot{q})\right), g_{t,i} = g_{t,i}\left(\delta_{i,n}(q), \delta_{i,t}(q)\right)$, $\phi_{t,i} = \phi_{t,i}\left(\delta_{i,n}(q), \delta_{i,t}(q), \dot{\delta}_{i,n}(q,\dot{q}), \dot{\delta}_{i,t}(q,\dot{q})\right)$ and fcm_i, Fc_i are the contact forces acting at the i'th contact point given with respect to the contact point's and the inertial frames respectively.

In abbreviated notation, it is required that for $\delta_{i,n} \geq 0$, $\{g_{n,i}, g_{t,i}, \phi_{n,i}, \phi_{t,i}\} \in \mathbb{R}$ it is both monotonically increasing, and continuous to its second derivative. The $\phi_{n,i}, \phi_{t,i}$ functions model the material's viscoelasticity property.
Hertzian based contact model, derived by (Walton,1978) is considered. Walton's model was derived for two identical linear-elastic, isotropic, perfectly adhesive spheres, load in a two dimensional linear oblique trajectory with a slope C, and it is here in the form:

$$fcm = \begin{pmatrix} g_{n,i} \\ g_{t,i} \end{pmatrix} + \begin{pmatrix} \phi_{n,i} \\ \phi_{t,i} \end{pmatrix} = \begin{pmatrix} \frac{8\bar{G}\sqrt{\tilde{r}_i}}{3(1-\nu)}(\delta_{i,n})^{3/2} \\ \frac{16\bar{G}\sqrt{\tilde{r}_i}}{3(2-\nu)}\sqrt{\delta_{i,n}}\delta_{i,t} \end{pmatrix} + Kd_v \cdot \dot{\delta}_i \quad ; \quad \delta_{i,n} \geq 0 \tag{4.1}$$

Here $\bar{G}, \nu$ and $\tilde{r}_i$ are respectively the shear modulus, Poisson ratio, and the equivalent radius of curvature $\tilde{r}_i = \frac{r_1 \cdot r_2}{r_1 + r_2}$ (Shapiro, et al., 2013), and $Kd_v \cdot \dot{\delta}_i$ is the viscoelastic term. Frictional contact is an essential property, if to consider a tangential load. The Coulomb friction model is assumed; that is, the normal and tangential forces at the i'th contact point are *"in the friction cone"* defined by $|f_{i,t}| \leq \mu f_{i,n}$ with μ the *coefficient of friction*.

5. Dynamic Model

A second order dynamic grasp system with $(\cdot)_0, (\cdot)_g$ respectively notating the object's and the gripper's parameters, is of the form

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + \tilde{G}(q) = \tau(q,\dot{q})$$

$$D(q) = \begin{bmatrix} D_0 & 0 \\ 0 & D_g \end{bmatrix} \quad C(q,\dot{q}) = \begin{bmatrix} C_0 & 0 \\ 0 & C_g \end{bmatrix} \quad \tilde{G}(q) = \begin{pmatrix} \tilde{G}_0 \\ \tilde{G}_g \end{pmatrix} \quad \tau = \left(G - J^T\right)Fc + \tau_{ctrl}. \tag{5.1}$$

Here $q \in \mathbb{R}^n; n = n_0 + n_g$ is the time dependent vector of system's configuration parameters, $\{D(q), C(q,\dot{q})\} \in \mathbb{R}^{n\times n}$ are the *inertia* and *Coriolis matrices*, $\tilde{G}(q) \in \mathbb{R}^n$ is the body forces vector, and $\tau(q,\dot{q}) \in \mathbb{R}^n$ is the sum of the external generalized forces vector applied to the system and the grasp control vector. A state vector of the form $\xi = \left(\xi_1^T \quad \xi_2^T\right)^T = \left(q^T \quad \dot{q}^T\right)^T$ defines a state space, with equilibrium point $\xi^* = (\xi_1^*, 0)$ so that $\dot{\xi}(\xi^*) = 0$.
This state is linearized at its equilibrium to be

$$\dot{\xi} = \begin{pmatrix} \xi_2 \\ D(\xi_1)^{-1}\left(\tau(\xi_1,\xi_2) - C(\xi_1,\xi_2)\xi_2 - \tilde{G}(\xi_1)\right) \end{pmatrix} \approx \begin{bmatrix} 0_{n\times n} & I_n \\ -D^{-1}Kp & -D^{-1}Kd \end{bmatrix}\left(\xi - \xi^*\right) = A\left(\xi - \xi^*\right). \tag{5.2}$$

The stiffness and dumping matrices $\{Kp, Kd\} \in \mathbb{R}^{n\times n}$ are sums of the corresponding contact points local matrices $Kp = \sum_{i=1}^{k} Kp_i$, $Kd = \sum_{i=1}^{k} Kd_i$, with explicit close form in (Ohev-Zion,2015).

6. Grasp synthesis Methods

A grasp in equilibrium may lose its dynamic stability due to increased normal preload or large tangential force – at the contact points, (Shapiro,2013)(Ohev-Zion,2014). Avoiding the stability lose is therefore done by maintaining an equilibrium configuration in which these forces are minimized. It is obvious on the other hand that these forces cannot vanish to zero - in the present of an external force, as they are required to maintain the grasp. Grasp synthesis must further consider the object-gripper coupling that is the contact model. A grasp stability criterion is therefore defined and serves the two following algorithms.

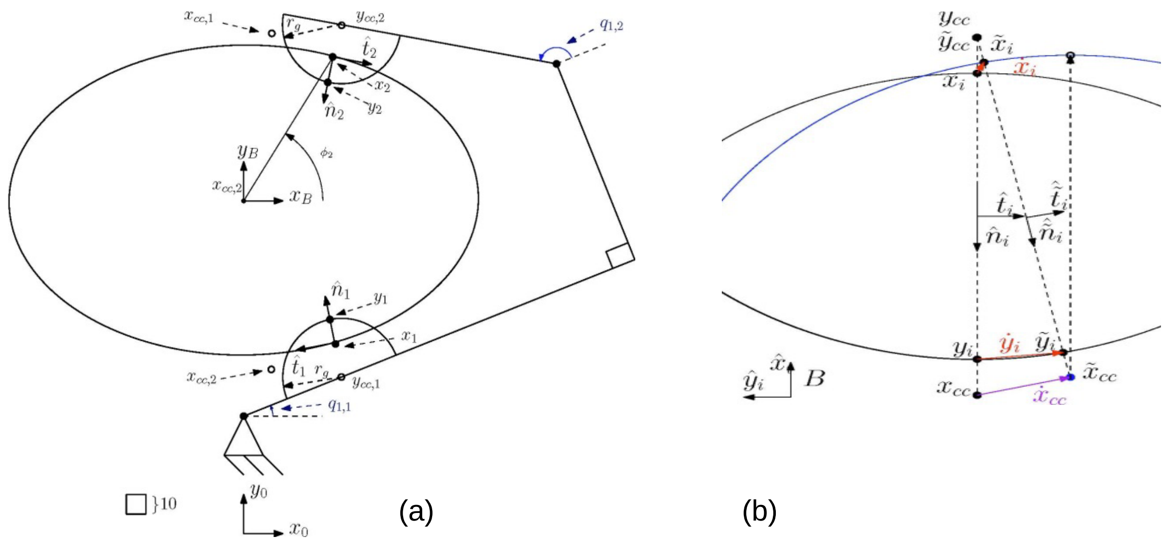

Figure 1: (a) Kinhematic grasp example and parameters, (b) change in Y_i due to change in q_0

The first is an offline algorithm that yields a gripper design with a grasp controller, which assures a stable grasp that matches the widest range of grasp configurations. It is innovative in the sense that no actual gripper is predefined, only the gripper's topology, so that the gripper's dimensions remains as variables for the algorithm to define. The algorithm generates parameters vectors, whose elements are the grippers' dimensions and their corresponding grasp controller gains. The parameters vectors of one set of k contact points' intersection with the other different sets, defines a gripper that best fit a stable grasp of the object and its appropriate controller gains. Processing this algorithm on variable objects, results with a gripper design that best matches multiple objects with varied stable grasp configurations (Sintov et.al.,2014).

Algorithm 6.1 - Gripper & Control Synthesis:

inputs:

an object's geometry, position, and orientation.

a gripper topology with k contact points, definition of its dimension's parameters vector, and the minimal and maximal values for each parameter

the maximal allowed contact force at a contact point $fcm_{i,max}$.

the minimal normal contact force fcm_{min}.

a grasp control structure, and its free parameters.

the external forces applied to the object.

begin:

for *all combinations of k contact points on the object:*

select a combination of k points on the object's circumference.

find $\min \sum_{i=1}^{k} fcm_i^2$ *under the constraints*

$$\sum Fc_i = 0, \sum M = 0, \left| fcm_{i,t} \right| \le \mu \, fcm_{i,n}, \; fcm_{i,n} \ge fcm_{i,min} > 0, \; fcm_i \le fcm_{i,max} \forall \{i;1 \ldots k\}$$

calculate the overlap segments' end points using the contact model.

for *all points of gripper's parameterized dimensions vector space*

find dimensions and configuration that provides the calculated contacts' positions.

find controller gains that guaranty a stable grasp - using the stability criterion.

end

end

intersect variables vectors sets per contact points

end

Walton's contact model is used and therefore the contact's load trajectories are considered to be linear $\delta_{i,t} = y_i(T) - y_i(0)$, so it satisfies the desired calculated force.

The second algorithm is performed online and provides a preferable grasp - based on a given gripper and system's configuration. Both of these algorithms can be thought of as inside-out processes that use the stability criterion in order to ensure a stable grasp and select the proper grasp controller.

Algorithm 6.2 – Grasp Planning:

inputs:

an object's geometry, position, and orientation from perception system

a gripper design with k contact points.

the maximal allowed contact force at a contact point $fcm_{i,max}$.

the minimal normal contact force fcm_{min} .

a grasp control structure, and its free parameters.

the external forces applied to the object.

begin:

***for** all combinations of k contact points on the object:*

select a combination of k points on the object's circumference.

find $\min \sum_{i=1}^{k} fcm_i^2$ under the constraints

$$\sum Fc_i = 0, \sum M = 0, \left| fcm_{i,t} \right| \leq \mu \, fcm_{i,n}, \, fcm_{i,n} \geq fcm_{i,min} > 0, \, fcm_i \leq fcm_{i,max} \forall \{i;1 \ldots k\}$$

calculate the overlap segments' end points using the contact model.

***if** exists a gripper's configuration in which the calculated contacts' positions are reachable.*

find controller gains that guaranty a stable grasp - using the stability criterion.

end

end

select configuration with the minimal fcm_i.

end

7. Example

The following presents the grasp's system's stability in equilibrium part of the algorithms. Implementing the criterion defined in section 2 to the dynamics derived in sections 3-5. Consider a rubber like elliptic object with major and minor radii $\{120,80\}[mm]$, grasped by an encircled ``finger'' – a configuration that is known as a *whole arm grasping*. The gripper's contacts have $30[mm]$ radius. The equilibrium configuration was found to be $d^* = (20,170)^T, \theta_0^* = 0$, $q_{1,1}^* = 2.6^\circ, q_{2,1}^* = 143.3^\circ$. A simplified PD grasp control is used to maintain the grasp $\tau_{ctrl} = (0_{3\times1}, \overline{\tau}_{ctrl})^T$, so that $\overline{\tau}_{ctrl} = \left(kps_{ctrl} I + kpas_{ctrl} \hat{J}\right)\left(q_g - q_g^*\right) + \left(kds_{ctrl} I + kdas_{ctrl} \hat{J}\right)\dot{q}_g$ $\{kps_{ctrl}, kpas_{ctrl}, kds_{ctrl}, kdas_{ctrl}\} \in \mathbb{R}$. The system's stability is assessed through the criterion for different values of controller gains – which defines a four dimensional parameter space. Cross sections of this space are presented in Figure 2.

The system's stability is shown to be more sensitive to changes in the anti-symmetric gains. An interesting outcome is the symmetry of the stability regions about the $kpas_{ctrl}$ axis, versus the asymmetry about $kdas_{ctrl}$ axis – with the area center shift from the origin, Figure 2(a). The cause for these shift and asymmetry is that the contact points are not symmetric with respect to the object's major axis, and therefore obtain different values for the normal and tangential local forces. It is seen in the dynamics explicit solution that the anti-symmetric part of Kp is directly affected by that difference in the tangential force, unlike the anti-symmetric part of Kd (Ohev-Zion,2015). The emerged interesting conclusion is that for some grasp configurations, it necessary to increase the system's anti-symmetric part in order to bring it to stable equilibrium. Further, by increasing the configuration point's distance from the edges of stability regions, the system's stability becomes more robust with respect to changes in the examined parameter. In this case the controller's gains visualization provides a controller design tool, for considering these parameters' influence to the system's stability together with other grasp properties such as robustness, applied force, of control effort.

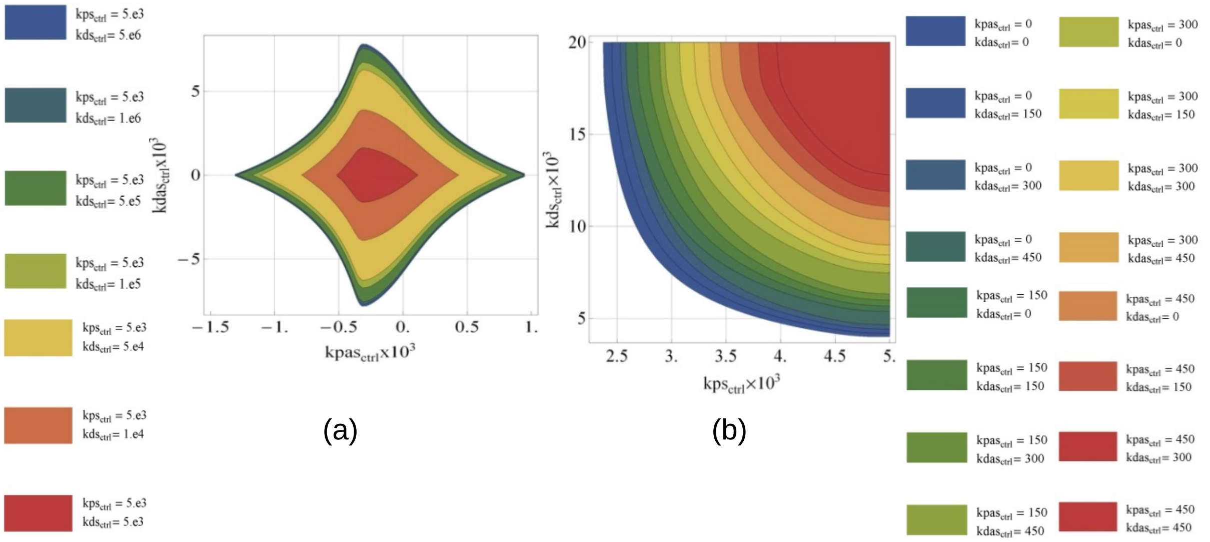

Figure 2: cross sections of stability regions, with respect to controller's gains

8. Discussions & Conclusions

A Stability criterion for anti-symmetric second order dynamic systems was derived, based on the linearized system's symmetry and anti-symmetry properties. Generic closed form kinematic and dynamic models were presented, which are compatible with varied grasp configuration – e.g. pinch, whole arm and combination of both. The models are further compatible for varied contact models – which comply with the current kinematic definition of δ_i . The dynamic grasp model's stability was analyzed at its equilibrium using the defined criterion. It provided an interesting observation that increasing the magnitude of the system's anti-symmetric parts, may improve\bring the system to stability.
Two innovative discrete grasp syntheses and planning were presented. The first provides a gripper and control design, which matches the vastest rang of grasping options. The second yields a minimal contact force grasp, for a given gripper and object configuration.

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10. References

- Ohev-Zion, A. (2015), Grasping of Deformable Objects, PhD thesis – Not publis*hed, Ben-Gurion University of the Negev, Mechanical Engineering Department, ISRAEL.*
- Ohev-Zion, A., Shapiro, A.,(2011) Grasping of Deformable Objects, Applied to Organic Produce*. Towards Autonomous Robotic Systems (TAROS), Lecture Notes in Computer Science(LNAI), 6856, (396-397)*, DOI: 10.1007/978-3-642-23232-9_45.
- Rimon E., Burdick, J.W., and Omata, T.(2006). A Polyhedral Bound on the Indeterminate Contact Forces in Planar Quasi-Rigid Fixturing and Grasping Arrangements. *IEEE Transactions on Robotics. 22. (240-255).* DOI: 10.1109/TRO.2005.862478.
- Shapiro, A.,(2005), Stability of Second-Order Asymmetric Linear Mechanical Systems With Application to Robot Grasping. *Journal of Applied Mechanics. 72. (966-968).* DOI:10.1115/1.2042484.
- Shapiro, A., Rimon, E., and Ohev-Zion, A.,(2013) On the Mechanics of Natural compliance in Frictional Contacts and its Effect on Grasp Stiffness and Stability. *The International Journal of Robotics Research. 32. (425-445).* DOI: 10.1177/0278364912471690.
- Sintov, A. Menassa, R. J. & Shapiro, A.,(2014), OCOG: A common grasp computation algorithm for a set of planar objects, *Robotics and Computer-Integrated Manufacturing , 30, (124-141)*
- Walton, K.,(1978). The oblique compression of two elastic spheres. *Journal Mech. Phys. Solids. 26.(139-150).*